RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

B.A./B.Sc. FIFTH SEMESTER EXAMINATION, DECEMBER 2015

THIRD YEAR [BATCH 2013-16]

Date : 18/12/2015 **PHYSICS (Honours)**

Time : 11 am – 1 pm **Paper : V [Group – B]** Full Marks : 50

[Use a separate Answer Book for each unit]

Unit – I

Answer **any three** questions

1. Answer the following questions: (5×2)
   a) Consider the electron double slit experiment with a monochromatic photo source to determine which slit the electron went through. If the intensity of the photo source is lowered keeping the frequency fixed, does the interference pattern reappear? What if the frequency of the photons is lowered, keeping the intensity fixed?
   b) Write down the *time-dependent* normalizable wave function for a free particle in one dimension.
   c) Give a sketch of the energy gap between the nth and the $(n-1)$ th energy levels of the particle in a one dimensional infinite potential well as a function of n.
   d) Why are observables in quantum mechanics chosen to be represented by Hermitian linear operators?
   e) If a system does not have stationary quantum states, what property of the Hamiltonian would you attribute this to?
2. Starting with the definition of the ground state of a one dimensional simple harmonic oscillator in terms of the annihilation operator $\hat{a}=\left[\frac{\hat{p}}{(2m\hbar\omega)^{1/2}}-i\left(\frac{m\omega}{2\hbar}\right)^{1/2}\hat{x}\right]$ which annihilates the state, obtain the normalized position space wave function for the ground and first excited states of the system. (5+5)
3. a) Determine the time derivatives of the expectation values of position and momentum operators and show that they obey classical laws. (5)
   b) Determine the energy levels and normalized wave functions of a particle of mass m in a box with rigid walls with $V(x)=\begin{cases}0 & \text{if } 0\le x\le a\\ \infty, & \text{otherwise}\end{cases}$. (5)
4. This question concerns the Hydrogen atom.
   a) Compare the trajectory of the earth around the sun with the ground state spatial probability distribution of the hydrogen atom. Is there any difference? Can you think why this difference occurs?
   b) Hydrogen atom spectral lines have frequencies obeying the Ritz Combination Principle: $\omega_{mn}=\omega_{mp}+\omega_{pm}$, $m<p<n$. Show that this follows from the energy spectrum of the hydrogen atom.
   c) Show that every energy level of the hydrogen atom with energy E_n has a total degeneracy of n^2. (3+4+3)
5. a) Use the uncertainty principle to estimate the ground state energy of hydrogen atom. (5)
   b) Find the commutation relation $[\hat{L}_x,\hat{L}_y]$ where $\hat{L}_x,\hat{L}_y$ are the operators for the components of the angular momentum $\hat{\vec{L}}$. Show that $\hat{L}^2$ commutes with all, $\hat{L}_x$, $\hat{L}_y,\hat{L}_z$. (5)

Unit – II

Answer **any two** questions

6. a) Determine the energy eigenvalues of the stationary states of a hydrogen like atom with atomic number z by using Bohr's theory. Mention all the assumptions. How does this result differ from the case of pure hydrogen atom. (5)

   b) An electron in an atom has an orbital angular momentum $\vec{l}$ and a spin angular momentum $\vec{s}$. Obtain an expression for the magnetic moment of the atom in terms of the Lande g-factor. If an atom is in the $^6G_{3/2}$ state, show that its magnetic moment is zero. (3)

   c) A beam of atoms in the $^2S_{1/2}$ state passes through a Stern-Gerlach setup. Into how many beams will it split? (2)

7. a) Consider the Lyman line $2P \rightarrow 1S$ of a hydrogen-like ion with atomic number z. With spin-orbit interaction the spectrum exhibits a doublet structure. The difference is wavenumber of the two emission lines is 29.6 cm^{-1}. Identify the ion. (3)

   [Given the doublet splitting of the (nl)-line to be

   $$\Delta U = \frac{(\alpha Z)^4}{2n^3 l(l+1)} E_0,\ \alpha = \frac{1}{137},\ E_0 = mc^2 = .511 \text{ Mev},\ hc = 1240\,\text{fm-Mev}]$$

   b) Consider the spectral lines given by the transitions:

   (a) $^1D_2 \rightarrow {}^1P_1$ of neutral Cadmium atom

   and (b) $^2P_{1/2} \rightarrow {}^2S_{1/2}$ (D_1)

   (c) $^2P_{3/2} \rightarrow {}^2S_{1/2}$ (D_2)

   of Na atom.

   When placed in a uniform magnetic field B, into how many sublevels will each of the states split? Show by means of energy level diagrams the allowed transitions in the three cases and the number of spectral lines that can be observed.

   Which of the atoms Cd and Na exhibit the anomalous Zeeman effect? (6+1)

8. a) Which of the following molecules will show rotational spectrum. Explain your answer:

   (a) CO, (b) CO_2 (c) OCS (d) N_2 (e) O_2 & (f) HCl.

   If one hydrogen atom in a hydrogen molecule is replaced by a deuterium atom, will there be any change in rotational constant and interatomic separation? Justify your answer. (3+2)

   b) What are the conditions a molecular vibration need to satisfy to be Raman active?

   What are the inadequacies of classical theory for explanning Raman effect.

   On what factors does the intensity of a Raman line depend? (2+2+1)

9. a) A system of 2-level atoms interacts with radiation at temperature T in a closed cavity. Discuss the various modes of energy exchanges in terms of Einstein's A and B coefficients.

   Which of the above modes can give rise to laser action? Explain. (4)

   b) i) At temperature T, let N_1 and N_2 be the number density of atoms in the lower state (energy E_1) and higher state (energy E_2) respectively. Radiation of energy density $u(\nu)$ at frequency $\nu = \frac{E_2 - E_1}{h}$ interacts with the atoms. Estimate the net stimulated emission energy per unit time per unit volume.

   ii) Hence argue that to have lasing action $N_2 > N_1$. How can this be achieved? Illustrate this schematically using the He–Ne laser. (3+3)

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